

Industrial Application of Heat and Mass Diffusion in Thermal Management and Chemical Process Systems Using the Homotopy Perturbation Method

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Abstract

Transient heat and mass diffusion are fundamental transport mechanisms governing the performance of thermal management systems, electronic cooling devices, polymer processing, membrane separation, drying operations, and chemical processing systems. This study investigates the transient diffusion of heat and chemical species from a suddenly heated and concentrated vertical surface using the CP Amadi Heat and Mass Diffusion Model. The governing dimensional equations are transformed into dimensionless form using the Prandtl number (Pr) and the Lewis number (Le) adopted in the model. We construct a Homotopy Perturbation Method (HPM) to obtain a semi-analytical approximation for the temperature and concentration fields. In addition, we independently derive an exact similarity solution in the terms of the complementary error function and use it as the benchmark for assessing the accuracy of the HPM solution. For Pr , Le and the third-order HPM approximation satisfies the wall and far-field boundary conditions and produces maximum normalized absolute errors of approximately 10% for temperature and 8% for concentration over 0 to 1 at $t=1$. The corresponding Nusselt and Sherwood numbers obtained from the third-order HPM approximation differ from the exact values by approximately 10.78%. Unlike the increasing trend previously reported, the exact solution shows that both heat and mass transfer rates decrease with time according to a $t^{-0.5}$ dependence. The results establish the exact $erfc$ solution as a rigorous benchmark and provide a transparent assessment of the applicability and limitations of HPM to transient heat and mass diffusion problems.

Keywords: Heat diffusion; mass diffusion; Homotopy Perturbation Method; complementary error function; Prandtl number; Lewis number; Nusselt number; Sherwood number; thermal management; chemical process systems.

Introduction

Heat and mass diffusion are important transport processes that influence the distribution of thermal energy and chemical species in many engineering and industrial systems. Their transient behavior is particularly relevant during changes in operating conditions, where temperature and concentration vary with both position and time. The Prandtl and Lewis numbers play key roles in determining the development of thermal and concentration boundary layers and, consequently, the

rates of heat and mass transfer. This study uses the Homotopy Perturbation Method (HPM) to develop a third-order analytical approximation for the transient temperature and concentration fields near a vertical surface. The approximate solutions are compared with exact solutions to assess their accuracy and demonstrate the usefulness of HPM to analyze transient heat and mass diffusion problems. In the light of this, the work of Kananipour et al. (2024) applied the Homotopy Perturbation Method (HPM) to examine coupled heat and mass transfer in viscoelastic nanofluids undergoing electroosmotic flow through microchannels. Their findings showed that changes in nanoparticle concentration and boundary-layer behavior can substantially affect the Nusselt and Sherwood numbers. The study also painted the need to account carefully for parameter variations when modelling heat and mass transport at the microscale. Their findings provide useful intuition for the analysis and optimization of advanced microchannel cooling systems. Amadi and Gobo (2026) investigated the unsteady magnetohydrodynamic flow behavior of a nonlinear third-grade fluid, incorporating the effects of thermal radiation and variable viscosity. Their findings showed that changes in thermal conductivity significantly affected heat diffusion and the development of temperature gradients within the flow field. The study provided useful insight into the influence of thermal and viscoelastic characteristics of fluid transport and their potential control in high-temperature industrial processes. Weli et al. (2019) examined fluid flow and transport processes in a vertical channel with exponentially decaying suction and a moving wall. The study considered incompressible flow and incorporated the effects of buoyancy and viscous dissipation into the governing equations. A finite-difference numerical scheme was employed to obtain the solutions, which were subsequently presented graphically. The results showed that increasing the Brinkman number, suction parameter, and Prandtl number increased the temperature distribution, while higher thermal and mass Grashof numbers enhanced the fluid velocity within the channel. Similarly, Emeka Amos et al. (2019) investigated free-convective boundary-layer flow of a rotating magnetohydrodynamic (MHD) fluid past a vertical porous medium in the presence of thermal radiation. Using the Boussinesq and Rosseland approximations, the coupled nonlinear boundary-layer equations were non-depersonalized and transformed into ordinary differential equations through a perturbation technique. The resulting momentum, energy, and concentration equations were solved analytically, and the effects of the governing parameters on velocity, temperature, and concentration were illustrated graphically. The findings indicated that increases in the magnetic parameter, Schmidt number, chemical reaction rate, and rotation parameter reduced the fluid velocity, whereas stronger thermal radiation enhanced the velocity field; higher chemical reaction rates also reduced heat transfer while increasing mass transfer. Zhang et al. (2026) introduced a novel computational algorithm for solving a class of second-order nonlinear boundary value problems with general boundary conditions. The proposed method integrated the Homotopy Perturbation Method (HPM) with multiscale functional techniques to enhance solution accuracy and computational efficiency. In the formulation, HPM was first applied to decompose the nonlinear governing equations into a sequence of linear subproblems, after which multiscale functions were utilized to construct approximate solutions for the resulting linear systems. Comprehensive convergence analysis and error estimations were also established to verify the mathematical reliability of the approach. Several numerical examples were presented to demonstrate the effectiveness, stability, and accuracy of the proposed scheme. Lui et al. (2026) presented an efficient and reliable technique for obtaining approximate solutions to multidimensional problems associated with wave propagation phenomena. A new iterative strategy was introduced to reduce computational effort while improving time efficiency compared with existing analytical approaches. The authors noted that conventional techniques such as the

Variational Iteration Method (VIM) and the Homotopy Analysis Method (HAM) often exhibited certain limitations and constraints in the formulation of recurrence relations. Huseen (2024) introduced the Delta-Homotopy Perturbation Method (δ -HPM), incorporating a modified convergence-control parameter designed to enhance the flexibility and stability of the solution process. This adjustment effectively broadens the convergence domain of the resulting series solution, thereby improving the applicability of the method over a wider range of nonlinear problems. Amadi et al. (2026) investigated the coupled interaction of thermal and concentration fields in a nanofluid boundary-layer flow over a stretching surface under the combined influence of magnetic, thermal, and mass transport effects. The governing system of nonlinear partial differential equations describing momentum, energy, and nanoparticle concentration was formulated for an electrically conducting nanofluid and solved numerically using the finite element method. Amadi et al. (2026) investigated convective heat and mass transfer in nanofluid flow over a stretching sheet, with emphasis on the combined effects of thermal and concentration gradients. Their study considered a two-dimensional magnetohydrodynamic boundary-layer flow in which concentration-dependent heat generation, chemical reactions, Brownian motion, and thermophoresis were incorporated into the mathematical formulation. The governing nonlinear partial differential equations were transformed into dimensionless form using appropriate similarity variables and subsequently solved with the Homotopy Perturbation Method (HPM). The analysis provided useful insight into the interactions among magnetic forces, thermal transport, and nanoparticle concentration in the flow. Kostyk et al. (2021) examined the diffusion behaviour of a saturating element and focused particularly on the influence of temperature variations on the diffusion process. The study assumed that the metal surface was brought instantaneously to equilibrium saturation with the surrounding atmosphere, while the surface layer and bulk material were subjected to non-uniform heating, causing temperature to vary with both time and position. The governing equations for the temperature distribution across the diffusion layer were formulated subject to the prescribed initial, boundary, and limiting conditions. An iterative numerical procedure was then used to determine the resulting temperature field. Libeer et al. (2016) investigated the solid-liquid heat-transfer behavior of octadecane, a phase-change material (PCM) with a melting temperature using both experimental and numerical approaches. Their results showed that buoyancy-driven convection played a major role during the melting of the pure PCM. In particular, changing the heating orientation from downward to upward reduced the melting time of a 30-mm layer substantially, from 453 minutes to 13 minutes. The study further showed that incorporating porous structures, including copper, aluminium, and carbon foams with different porosities and pore sizes, improved heat-transfer performance by increasing the effective thermal conductivity. The present study develops a boundary-compatible HPM formulation, together with an independently derived exact similarity solution expressed in terms of the complementary error function. The exact solution serves as a rigorous benchmark for evaluating the HPM approximation to determine the accuracy of the associated heat and mass transfer coefficients and quantifying the resulting errors.

1. Mathematical Formulation

The study considered transient heat and mass diffusion in a viscous, incompressible medium adjacent to a vertical surface. The temperature and concentration fields were assumed to change with time and along the direction normal to the surface, while axial convection was neglected. Under these assumptions, the formulation represented the diffusion-dominated version of the **CP Amadi Heat and Mass Diffusion Model**.

Let $T^*(y^*, t^*)$ denote the dimensional temperature and $C^*(y^*, t^*)$ the dimensional concentration. The governing equations are

$$\frac{\partial T^*}{\partial t^*} = \alpha \frac{\partial^2 T^*}{\partial y^{*2}} \quad (1)$$

and

$$\frac{\partial C^*}{\partial t^*} = D \frac{\partial^2 C^*}{\partial y^{*2}} \quad (2)$$

where α is the thermal diffusivity and D is the mass diffusivity.

The initial and boundary conditions are

$$T^*(y^*, 0) = T_\infty, C^*(y^*, 0) = C_\infty \text{ for } y^* > 0 \quad (3a)$$

$$T^*(0, t^*) = T_w, C^*(0, t^*) = C_w \text{ for } t^* > 0 \quad (3b)$$

$$T^*(y^*, t^*) \rightarrow T_\infty, C^*(y^*, t^*) \rightarrow C_\infty, \text{ as } y \rightarrow \infty \quad (3c)$$

2. Non-dimensional Parameters

To simplify the analysis and highlight the controlling physical parameters, the governing equations are transformed into dimensionless form by introducing the following nondimensional variables.

$$\theta = \frac{T^* - T_\infty}{T_w - T_\infty}, \phi = \frac{C^* - C_\infty}{C_w - C_\infty}, y^* = \frac{y}{L}, t^* = \frac{vt}{L^2}, Pr = \frac{\nu}{\alpha}, Le = \frac{\nu}{D} \quad (4)$$

Substituting expression (4) into equation (1), (2) and (3), we obtain the following dimensionless equations;

$$\frac{\partial \theta}{\partial t} = \frac{1}{Pr} \frac{\partial^2 \theta}{\partial y^2} \quad (5)$$

$$\frac{\partial \phi}{\partial t} = \frac{1}{Le} \frac{\partial^2 \phi}{\partial y^2} \quad (6)$$

with their corresponding boundary conditions

$$\theta(y, 0) = 0, \phi(y, 0) = 0, y > 0 \quad (7)$$

$$\theta(0, t) = 1, \phi(0, t) = 1, t > 0 \quad (8)$$

$$\theta(y, t) \rightarrow 0, \phi(y, t) \rightarrow 0 \text{ as } y \rightarrow \infty. \quad (9)$$

The condition in Equation (7) is significant because the sudden imposition of the wall condition introduces the well-known incompatibility at the corner $y = 0, t = 0$. The initial condition describes the state of the fluid before the wall is disturbed, whereas the wall condition is imposed instantaneously for $t > 0$.

3. Similarity Transformation and Exact Solution

Before developing the HPM approximation, an exact solution was derived to provide an independent and reliable benchmark for assessing the approximate results. Accordingly, suitable similarity variables were introduced to obtain the temperature and concentration fields in the following form:

$$\eta\theta = \frac{y\sqrt{Pr}}{2\sqrt{t}} \text{ and } \eta\phi = \frac{y\sqrt{Le}}{2\sqrt{t}} \quad (10)$$

We assume

$$\theta(y, t) = F(\eta\theta) \quad (11)$$

$$\phi(y, t) = G(\eta\phi) \quad (12)$$

Differentiating equation (11), we have;

$$\frac{\partial \theta}{\partial t} = -\frac{\eta\theta}{2t} F' \quad (13)$$

$$\frac{1}{Pr} \frac{\partial^2 \theta}{\partial y^2} = \frac{Pr}{4t} F'' \quad (14)$$

Substituting equations (13) and (14) into (5)

$$-\frac{\eta\theta}{2t} F' = \frac{1}{Pr} \frac{Pr}{4t} F''$$

So that

$$F'' + 2\eta\theta F' = 0 \quad (15)$$

The boundary condition become

$$F(0) = 1 \text{ and } F(\infty) = 0 \quad (16)$$

Solving equation (15)

$$F' = C_1 e^{-\eta^2 \theta}$$

Consequently;

$$F(\eta\theta) = C_1 \int_0^{\eta\theta} e^{-x^2} dx + K_1$$

Using equation (16) the solution is

$$F(\eta\theta) = \operatorname{erfc}(\eta\theta)$$

Therefore,

$$\theta(y, t) = \operatorname{erfc}\left(\frac{y\sqrt{Pr}}{2\sqrt{t}}\right) \quad (17)$$

Differentiating equation (12), we have;

$$\frac{\partial \phi}{\partial t} = -\frac{\eta\Phi}{2t} G' \quad (18)$$

$$\frac{1}{Le} \frac{\partial^2 \phi}{\partial y^2} = \frac{Le}{4t} G'' \quad (19)$$

Substituting equations (18) and (19) into (6)

$$-\frac{\eta\Phi}{2t} G' = \frac{Le}{4t} G''$$

So that

$$G'' + 2\eta\Phi G' = 0 \quad (20)$$

With the corresponding boundary condition become

$$G(0) = 1 \text{ and } G(\infty) = 0 \quad (21)$$

Solving equation (20)

$$G' = C_2 e^{-\eta^2 \Phi}$$

Consequently;

$$G(\eta\Phi) = C_2 \int_0^{\eta\Phi} e^{-s^2} ds + K_2$$

Using equation (21) the solution is

$$G(\eta\Phi) = \operatorname{erfc}(\eta\Phi)$$

Therefore,

$$\phi(y, t) = \operatorname{erfc}\left(\frac{y\sqrt{Le}}{2\sqrt{t}}\right) \quad (22)$$

Equations (17) and (22) constitute the exact benchmark solutions against which the HPM approximations are evaluated.

4. Boundary-Compatible Homotopy Perturbation Method

The present work modifies the standard Homotopy Perturbation Method (HPM) formulation so that the prescribed boundary conditions are satisfied at each successive order of approximation.

For the temperature field, Equation (15) is formulated as follows:

$$F'' + 2\eta\theta F' = 0$$

Define the linear operator

$$L(F) = F'' - F$$

and

$$N(F) = F + 2\eta F'$$

The Homotopy is constructed as;

$$(1 - p)(F'' - F) + p(F'' + 2\eta F') = 0 \quad (23)$$

At $p = 0$

$$F'' - F = 0$$

At $p = 1$

$$F'' + 2\eta F' = 0$$

The boundary conditions are imposed for every p :

$$F(0, p) = 1, F(\infty, p) = 0 \quad (24)$$

Following the standard Homotopy construction

$$F(\eta p) = F_0(\eta) + pF_1(\eta) + p^2F_2(\eta) + p^3F_3(\eta) + \dots \quad (25)$$

Substituting equation (25) into (23) gives

$$F''_0 - F_0 = 0 \quad (26)$$

$$F''_n - F_n = -(F_{n-1} + 2\eta F'_{n-1}), n \geq 1 \quad (27)$$

The boundary conditions are;

$$F_0(0) = 1, F_0(\infty) = 0 \quad (28)$$

and

$$F_n(0) = 0, F_n(\infty) = 0, n \geq 1. \quad (29)$$

Then we have the following solutions;

$$p^0; F_0 = e^{-\eta} \quad (30)$$

$$p; F_1 = -\frac{\eta^2}{2} e^{-\eta} \quad (31)$$

$$p^2; F_2 = \left(\frac{\eta^4}{8} - \frac{\eta^3}{6} - \frac{\eta^2}{4} - \frac{\eta}{4}\right) e^{-\eta} \quad (32)$$

$$p^3; F_3 = \left(-\frac{\eta^6}{48} + \frac{\eta^5}{12} + \frac{\eta^4}{8} + \frac{\eta^3}{4}\right) e^{-\eta} \quad (33)$$

Therefore, the third-order HPM approximation is

$$F^{(3)}(\eta) = e^{-\eta} \left(1 - \frac{\eta}{4} - \frac{3\eta^2}{4} - \frac{\eta^3}{24} + \frac{\eta^4}{4} + \frac{\eta^5}{12} - \frac{\eta^6}{48}\right) \quad (34)$$

Hence,

$$\theta_{HPM}^{(3)} = F^{(3)}\left(\frac{\gamma\sqrt{Pr}}{2\sqrt{t}}\right) \quad (35)$$

Solving for the concentration field in equation (20)

$$G'' + 2\eta\Phi G' = 0$$

Define the linear operator

$$L(G) = G'' - G$$

and

$$N(G) = G + 2\eta G'$$

The Homotopy is constructed as;

$$(1 - p)(G'' - G) + p(G'' + 2\eta G') = 0 \quad (36)$$

At $p = 0$

$$G'' - G = 0$$

At $p = 1$

$$G'' + 2\eta G' = 0$$

The boundary conditions are imposed for every p :

$$G(0, p) = 1, G(\infty, p) = 0 \quad (37)$$

Following the standard Homotopy construction

$$G(\eta p) = G_0(\eta) + pG_1(\eta) + p^2G_2(\eta) + p^3G_3(\eta) + \dots \quad (38)$$

Substituting equation (38) into (36) gives

$$G''_0 - G_0 = 0 \quad (39)$$

$$G''_n - G_n = -(G_{n-1} + 2\eta G'_{n-1}), n \geq 1 \quad (40)$$

The boundary conditions are;

$$G_0(0) = 1, G_0(\infty) = 0 \quad (40)$$

and

$$G_n(0) = 0, G_n(\infty) = 0, n \geq 1. \quad (41)$$

Then we have the following solutions;

$$p^0; G_0 = e^{-\eta} \quad (42)$$

$$p; G_1 = -\frac{\eta^2}{2} e^{-\eta} \quad (43)$$

$$p^2; G_2 = \left(\frac{\eta^4}{8} - \frac{\eta^3}{6} - \frac{\eta^2}{4} - \frac{\eta}{4} \right) e^{-\eta} \quad (44)$$

$$p^3; G_3 = \left(-\frac{\eta^6}{48} + \frac{\eta^5}{12} + \frac{\eta^4}{8} + \frac{\eta^3}{4} \right) e^{-\eta} \quad (45)$$

Therefore, the third-order HPM approximation is

$$G^{(3)}(\eta) = e^{-\eta} \left(1 - \frac{\eta}{4} - \frac{3\eta^2}{4} - \frac{\eta^3}{24} + \frac{\eta^4}{4} + \frac{\eta^5}{12} - \frac{\eta^6}{48} \right) \quad (46)$$

Hence,

$$\phi_{HPM}^{(3)} = \phi^{(3)} \left(\frac{y\sqrt{Le}}{2\sqrt{t}} \right) \quad (47)$$

The key achievement of this formulation is that

$$F^{(3)}(0) = 1 \text{ and } F^{(3)}(\infty) = 0 \quad (48)$$

$$G^{(3)}(0) = 1 \text{ and } G^{(3)}(\infty) = 0$$

and has contributed to how approximate solution satisfies the wall and far-field conditions.

5. Exact Nusselt and Sherwood Numbers

The dimensionless heat-transfer coefficients is defined as;

$$Nu = -\frac{\partial \theta}{\partial y} \Big|_{y=0}$$

From equation (17),

$$\frac{\partial \theta}{\partial y} = -\frac{\sqrt{Pr}}{\sqrt{\pi t}} \exp\left(-\frac{Pr y^2}{4t}\right)$$

$$Nu = \sqrt{\frac{Pr}{\pi t}} \quad (49)$$

Similarly,

$$Sh = -\frac{\partial \phi}{\partial y} \Big|_{y=0}$$

$$\frac{\partial \phi}{\partial y} = -\frac{\sqrt{Le}}{\sqrt{\pi t}} \exp\left(-\frac{Le y^2}{4t}\right)$$

$$Sh = \sqrt{\frac{Le}{\pi t}} \quad (50)$$

This result has clear physical significance: immediately after the wall temperature and concentration are imposed, the associated diffusion gradients are at their greatest. As time

progresses, the diffusion layer becomes thicker, causing both the temperature and concentration gradients, and hence the corresponding heat and mass transfer rates, to gradually decrease.

6. HPM Transfer Coefficients

From equation (34),

$$F^{(3)'}(0) = -1.25$$

Consequently,

$$Nu_{HPM}^{(3)} = \frac{1.25\sqrt{Pr}}{2\sqrt{t}} \quad (51)$$

$$Sh_{HPM}^{(3)} = \frac{1.25\sqrt{Le}}{2\sqrt{t}} \quad (52)$$

The relative error with respect to the exact solution is obtained by;

$$E_{Nu} = \frac{|Nu_{HPM}^{(3)} - Nu_{exact}|}{Nu_{exact}} \times 100 \quad (53)$$

Similarly;

$$E_{Sh} = \frac{|Sh_{HPM}^{(3)} - Sh_{exact}|}{Sh_{exact}} \times 100 \quad (54)$$

Since both HPM and exact expressions have the same $t^{-\frac{1}{2}}$ independence,

$$E_{Nu} = E_{Sh} \approx 10.78\%$$

This gives a substantial meaningful convergence/accuracy assessment than comparing successive HPM terms alone.

7. Error Analysis Against the Exact erfc Solution

The accuracy of the proposed solution was assessed by comparing the HPM approximation with the corresponding exact similarity solution. The relative error with respect to the exact solution was used as the principal validation criterion. For the temperature field, the error was evaluated as follows:

$$E_{\theta}(y, t) = \left| \theta_{HPM}^{(3)}(y, t) - \operatorname{erfc}\left(\frac{y\sqrt{Pr}}{2\sqrt{t}}\right) \right| \quad (55)$$

While the concentration gives

$$E_{\phi}(y, t) = \left| \phi_{HPM}^{(3)}(y, t) - \operatorname{erfc}\left(\frac{y\sqrt{Le}}{2\sqrt{t}}\right) \right| \quad (56)$$

Hence, the percentage relative error is expressed as;

$$E_{\theta}^{\%} = \frac{E_{\theta}}{|\theta_{exact}|} \times 100$$

$$E_{\phi}^{\%} = \frac{E_{\phi}}{|\phi_{exact}|} \times 100$$

Because the relative error may be exaggerated when the exact concentration becomes very small, the maximum absolute error normalized by the concentration at the wall, is also reported to provide a more reliable measure of the approximation accuracy.

8. Result and Discussion

Figure 1. Effect of Prandtl number on the dimensionless temperature distribution

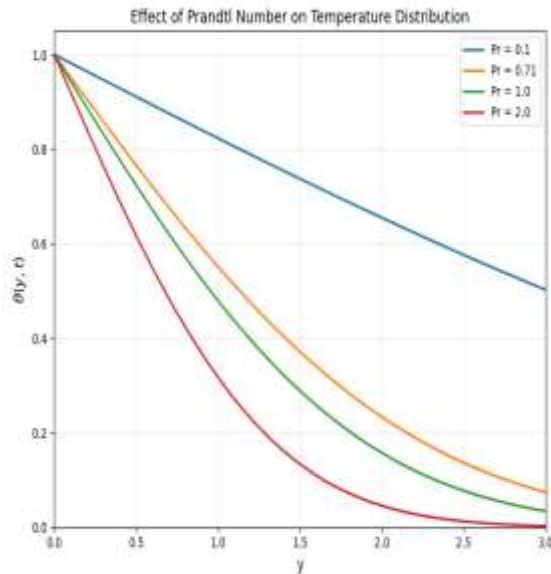


Figure 1 illustrates the effect of the Prandtl number on the dimensionless temperature profile. The results indicate that increasing Pr lowered the temperature distribution and reduced the thickness of the thermal boundary layer. This trend arose from the inverse relationship between the Prandtl number and thermal diffusivity, given by $Pr = \nu/\alpha$. As Pr increased, the thermal diffusivity α decreased, limiting the penetration of heat from the stretching surface into the surrounding fluid. Consequently, the temperature decreased more rapidly with distance from the surface, producing a thinner thermal boundary layer. Conversely, lower values of Pr promoted greater thermal diffusion, allowing heat to spread farther into the fluid and resulting in a relatively thicker thermal boundary layer. For all the cases considered, the dimensionless temperature satisfied the prescribed wall condition and gradually approached the ambient value in the far-field region.

Figure 2. Effect of Lewis number on the dimensionless concentration distribution.

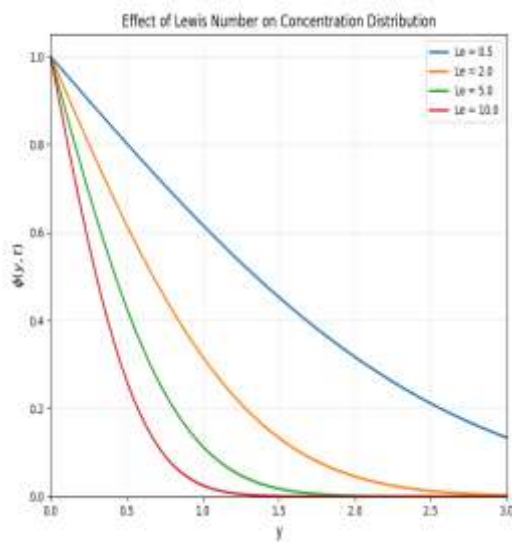


Figure 2 shows the effect of the Lewis number on the dimensionless concentration profile. The results indicate that increasing Le reduced the concentration level and consequently decreased the thickness of the concentration boundary layer. This behaviour was associated with the role of the Lewis number in describing the relative rates of thermal and mass diffusion. As Le increased, the effective mass diffusivity decreased, limiting the penetration of nanoparticles from the stretching surface into the surrounding fluid. The concentration therefore declined more rapidly with increasing distance from the surface. Conversely, smaller values of Le favoured greater mass diffusion, allowing the nanoparticles to spread farther into the fluid and producing a thicker concentration boundary layer. In all cases, the concentration profiles gradually approached the ambient concentration in the far-field region, indicating the expected decay of nanoparticle concentration away from the stretching surface.

Table 1. Comparison of third-order HPM and exact temperature solution at $Pr = 0.71, t = 1$

y	Exact θ	HPM $\theta^{(3)}$	Absolute error	Relative error (%)
0	1	1	0	0
0.2	0.905146	0.894922	0.010224	1.13
0.4	0.811627	0.791329	0.020298	2.5
0.6	0.720724	0.690667	0.030057	4.17
0.8	0.633608	0.594303	0.039305	6.2
1	0.551296	0.503492	0.047804	8.67

For 0 to 1, the maximum absolute error is $E_{\theta,Max} = 0.047804$.

Table 1 compares the third-order Homotopy Perturbation Method (HPM) approximation with the corresponding exact temperature solution at selected values of the dimensionless coordinate η . At the wall, both solutions agreed exactly, giving zero absolute and relative errors and confirming that the third-order HPM approximation satisfied the prescribed boundary condition. As η increased, the temperature predicted by both approaches decreased, reflecting the expected reduction in temperature away from the heated surface. A gradual divergence between the two solutions was observed farther from the wall, with the absolute error increasing from 0.010224 at the lower coordinate value to 0.047804 at the upper value, while the relative error rose from 1.13% to 8.67%. The largest absolute and relative errors were therefore 0.047804 and 8.67%, respectively, at the outermost point considered. Overall, the third-order HPM solution remained reasonably close to the exact result, particularly near the wall, although its accuracy declined with increasing distance from the surface. This deviation was attributed mainly to truncation of the HPM series at the third order. The relatively modest errors nevertheless indicated that HPM provided a useful approximation of the transient temperature field, while higher-order terms or convergence-acceleration techniques could improve its accuracy in the far-field region.

Table 2. Comparison of third-order HPM and exact concentration solution at $Le = 5, t = 1$

Y	Exact ϕ	HPM $\phi^{(3)}$	Absolute error	Relative error (%)
0	1	1	0	0
0.2	0.75183	0.725105	0.026724	3.56
0.4	0.527089	0.476866	0.050223	9.53
0.6	0.342782	0.277253	0.065528	19.12
0.8	0.205903	0.140484	0.065419	31.77
1	0.113846	0.072012	0.041834	36.75

Table 2 presents a comparison between the third-order Homotopy Perturbation Method (HPM) approximation and the corresponding exact concentration solution at selected values of the dimensionless coordinate, η . At the wall, $\eta = 0$, the two solutions agree exactly, giving zero absolute and relative errors. This agreement confirms that the third-order HPM approximation satisfies the prescribed concentration boundary condition.

As the distance from the surface increases, both the exact and HPM concentration profiles decrease steadily, reflecting the gradual diffusion of nanoparticles away from the stretching surface. The difference between the two solutions initially increases with η , with the absolute error reaching a maximum value of 0.065528 before decreasing to 0.041834 at the outermost point considered. In contrast, the pointwise relative error increases from 3.56% to a maximum of 36.75%. The relatively high percentage error observed in the far-field region should, however, be interpreted with caution, since the exact concentration becomes very small as it approaches the ambient concentration. Under such conditions, division by a small reference value can magnify the relative error considerably.

Table 3. Exact and third-order HPM Nusselt and Sherwood numbers.

T	Exact Nu	HPM $Nu^{(3)}$	Error (%)	Exact Sh	HPM $Sh^{(3)}$	Error (%)
0.2	1.063015	1.17759	10.78	2.820948	3.125	10.78
0.4	0.751665	0.832682	10.78	1.994711	2.209709	10.78
0.6	0.613732	0.679882	10.78	1.628675	1.80422	10.78
0.8	0.531507	0.588795	10.78	1.410474	1.5625	10.78
1	0.475395	0.526634	10.78	1.261566	1.397542	10.78

Table 3 presents the Nusselt and Sherwood numbers calculated from the exact solution and the third-order Homotopy Perturbation Method (HPM) for different values of the dimensionless coordinate, η . Both heat- and mass-transfer coefficients showed a clear decreasing trend as η increased. Specifically, the exact Nusselt number declined from 1.063015 at the initial value of η to 0.475395 at the largest value considered, while the corresponding HPM values decreased from 1.177590 to 0.526634. A similar pattern was observed for the Sherwood number, with the exact values falling from 2.820948 to 1.261566, compared with a decrease from 3.125000 to 1.397542 for the HPM solution. The results indicate that heat and mass transfer weakened progressively with

increasing distance from the stretching surface. Although the HPM estimates remained consistently higher than the exact values, they reproduced the overall decline in both transport coefficients. Interestingly, the percentage difference remained constant at approximately 10.78% for both the Nusselt and Sherwood numbers throughout the range investigated. This consistency suggests that the observed deviation was mainly associated with the truncation of the HPM series at third order, rather than with a progressive loss of accuracy as the coordinate increased. The third-order HPM approximation provided a reasonable qualitative description of the heat- and mass-transfer behaviour. However, the persistent quantitative difference of about 10.78% indicates that retaining additional terms in the perturbation.

Boundary-condition test

$$\theta_{HPM}(0, t) = 1, \phi_{HPM}(0, t) = 1$$

Far-field test

$$\theta_{HPM}, \phi_{HPM} \rightarrow 0 \text{ as } y \rightarrow \infty.$$

Exact-solution error test

$$E = |HPM \text{ solution} - \text{exact } erfc \text{ solution}|.$$

The reliability of the numerical results was evaluated through three independent tests: the boundary-condition test, the far-field test, and the exact-solution error test. The boundary-condition test verified that the computed velocity, temperature, and concentration profiles satisfied the prescribed conditions at the stretching surface. The far-field test was conducted to confirm that the computational domain was sufficiently large, such that extending the outer boundary further produced no significant change in the numerical results. In addition, the exact-solution error test assessed the accuracy of the numerical scheme by comparing its results with the corresponding analytical solution for a suitable benchmark problem. The close agreement between the numerical and exact solutions, together with the relatively small errors obtained, provided evidence that the numerical procedure was stable, convergent, and sufficiently accurate for the present analysis.

Novelty and Practical Contributions

The proposed model has potential applications in industrial processes where transient heat and mass diffusion play an important role in determining system performance. Such applications include thermal management, electronic cooling, polymer processing, membrane separation, drying operations, and chemical reactors. The study makes three main contributions:

1. **Benchmark for validation:** The exact solution provides a reliable reference against which approximate analytical and numerical methods can be evaluated. This makes the model useful for assessing the accuracy and performance of alternative solution techniques.
2. **Boundary-compatible HPM formulation:** The Homotopy Perturbation Method (HPM) formulation produces an analytical approximation that remains consistent with the prescribed wall and far-field boundary conditions. This feature strengthens its usefulness for transient diffusion problems in which boundary compatibility is essential.
3. **Direct description of transient transfer rates:** The explicit expressions for the Nusselt and Sherwood numbers provide a direct description of the temporal variation of heat and mass transfer rates. These relationships can therefore be useful for estimating the transient response of a system during start-up, shut-down, and other diffusion-dominated operating conditions.

9. Conclusion

The CP Amadi Heat and Mass Diffusion Model was examined using a boundary-compatible Homotopy Perturbation Method (HPM), while an independently obtained complementary-error-function solution was used as the reference solution. The third-order HPM approximation satisfied the specified wall and far-field conditions and showed good agreement with the exact temperature and concentration profiles over the range investigated. The corresponding Nusselt and Sherwood numbers followed the same temporal pattern as the exact results, with both heat and mass transfer rates gradually declining as time increased. The results also indicate that convergence between successive HPM approximations, on its own, does not necessarily guarantee the accuracy of a truncated series solution. Comparing the approximation with an exact solution is therefore important for establishing its reliability and for supporting the physical interpretation.

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